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How to Reduce Korean Unification Costs : Two-Region Endogenous Growth Model with Human Capital

남북한 통일비용을 줄이는 방안 :
인적자본 고려한 두 지역간 내생적 성장모형을 중심으로

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문 동 규

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이 논문을 경제학석사 학위논문으로 제출함.

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Abstract

How to Reduce Korean Unification Costs : Two-Region Endogenous Growth Model with Human Capital

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The purpose of this thesis is to estimate unification costs of North and South Korea and to obtain economic implications of how to reduce its costs. Costs are calculated by present value of dynamics of government transfer from the South to the North, using two-region endogenous growth model with human capital. Considering lessons from German reunification case and the North and South Korea's situation, I set up plausible three scenarios and dates for Korean unification. As a result, the North's reform and open market policy as well as the South's cooperative stance substantially reduce total costs of Korean unification.

Keywords : Korean unification costs, Endogenous growth model, North Korea and South Korea, Unification, Human capital.

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1 Introduction

After German reunification in October 1990, Korea is the last divided country in the world. For more than sis decades, since 1953, South Korea has developed fast and finally made economic miracle[1], on the other hand, North Korea has lagged far behind under Communism society. Therefore, GNI per capita of South Korea is about $25,000 but the North's is slightly over $1,300 in 2014[2]. Based on the fact that they share large portion of common denominator such as race, history, culture, natural sources, etc, this huge economic gap, today they are faced, is astonishing and could lead to considerable fiscal burden to the South Korean government when they are once unified.

Calculating unification costs, we should consider economic concept of 'net-costs': total costs - total benefits. If net costs are negative which means positive net benefits, it would be better to unify. If not, the other way around. However, It is difficult to measure benefits of unification which are far beyond economics field[3]. Moreover, they are also not much dependent of unification process since benefits will occur in the long period[4]. However, calculating costs are comparatively easy task and highly dependent upon unification process since much of costs occur in the short time. Therefore, considering benefits are given, reducing costs is equivalent to maximizing net benefits of unification. That is why we only focus on reducing costs in this paper.

[1]Lucas, Robert E. JR(1993) described Korean case as a miracle in his paper "Making a Miracle".

[2]Bank of Korea.

[3]Not only GDP, but also various psychological factors such as love for mankind, patriotism, pride, etc are would be main benefits for unification.

[4]KDI North Korean Economic Review(KDI 북한경제리뷰, 11월호), November, 2012

Unfortunately, previous studies about estimating unification costs are criticized today because of their wide variation of estimated costs according to researchers and analytical methods. Some major results are as follows. KDI(Korea Development Institute)(1994) estimated approximately from $980 billions to $1 trillions assuming unification in 2000. Noland et al.(1996) estimated from $415 billions to $983 billions assuming unification in 2000. Goldman Sachs estimated up to $3.5 trillions. SERI(Samsung Economic Research Institute)(2005) estimated about $546 billions as a itemized budget methodology. Beck(2010) calculated from $2 to $5 trillions to make North Korean income up to 80 percent of South Korean's over 30 years after unification[5]. Considering its highly speculative nature, wide spread of estimation of costs is not surprising. Besides, another problem is that many studies are based on simple comparative static analysis rather than dynamic analysis.

In this thesis, I estimated Korean unification costs by calculating present value of government transfer from the South to the North over 50 years after unification, making GDP per capita in the North to be equal to the South's. To do so, I mainly used modified and expanded version of model used in Funke and Strulik(2005). I also considered human capital, which is a key factor of economic growth, and three scenarios. Scenario 1 is German style unification : abrupt unification without sufficient preparation. Scenario 2 is economic cooperation unification : the South helps and cooperates with the North then unify. Scenario 3 is the North's Chinese style reform and open market policy then unification. Scenarios in Kim and Roland(2012) are referred in this paper. We will see more details in Chapter 4.

Organazation of this thesis is as follows. In Chapter 2, we will take a look into German reunification as a precious case study. In Chapter 3, the

[5]Song, Joonhyuk(2014). See more on Table 11 in Appendix I:Additional Data.

North and South's economy status is provided to grasp their status quo and to estimate some major parameters. In Chapter 4, we set up model to estimate unification costs depends on scenarios. In Chapter 5, calibrating parameters for computer calculating. In Chapter 6, graphical results are provided using Matlab program. In Chapter 7, we discuss results we get from Chapter 6 and obtain implications of how to reduce its costs. Chapter 8 is references and I provide Matlab code and additional data in Appendix.

2 Lessons from German Reunification Case

When the Berlin wall fell in 1989, many East Germans moved to the West showing their willingness to reunification so called 'vote with feet'[6]. Approximately a year later, Germany was officially reunified on October 3 1990. Today, Germany is rank 4 biggest country in the world in terms of nominal GDP[7] , the world-renowned manufacturing country and world-class high technology country. Underneath the bright surface, however, Germany is still suffering from aftermaths of reunification. In this case, what lessons can we draw from the German case?

First, actual reunification costs are far more than estimated costs before the reunification. Before monetary integration of the Deutsche Mark and the East Mark, German Economic research institutes estimated 50 billions DM for the West's government transfer annually and about 70 billions DM right before Monetary integration. However, actual government transfer is about 200 billions DM annually without decreasing until today. Such an continuous fiscal burden occurs two problems - one is tax resistance and social chronic fatigue for the West, the other is fading will of independence for the East.[8]

Second, in addition to the amount of fiscal burden, the composition of cost is more serious. About 50% of government transfer spend on social security expenditure for unemployed, elderly, children or poor East German[9]. This kind of government transfer is absolutely necessary to maintain healthy society but the problem is its degree - 50% of total expenditure it burdensome. This is why reunification costs has not reduced since 2000 and

[6]고일동(2010), 통일비용 논의의 허와 실, KDI,p13
[7]United Nations(2012)(http://unstats.un.org/unsd/snaama/dnltransfer.asp?fID=2)
[8]고일동(2010), 통일비용 논의의 허와 실, KDI, p6
[9]연방건설교통부 추정, 독일연방경제자문위원회(SVR, 2004) 제628항

has charged heavy burden on the West tax payers.

Third, compared to the German case, Korean economic status is more pessimistic. Intuitively and simply, unification costs are a function of gap of GDP. The bigger GDP gap between two countries, the more unification costs are needed. GNI per capita of West Germany was about 4 times of that of the East at that reunification time. But it is about 20 times in Korean case[10]. Beyond simple number of GDP, qualitative gap between the North and the South makes the situation worse. The East Germany was an industrialized country with open economy to the world and had high level of economic interchange with the West. On the other hand, North Korea is representative closed economy and one of the poorest countries in the world.

Fourth, careless monetary integration between the Deutsche Mark and the East Mark led to overnight loss of competativeness of East German industry and to massive unemployment which led to high amount of social security expenditure. Moreover, monetary integration was driven by politicians who are blinded by East German votes[11].

Fifth, insufficient preparation increased reunification costs. It took about an year to officially reunified after the Berlin wall fell. If they had more time to prepare in advance, they would have reduced costs considerably. Luckily and unluckily at the same time, this kind of event-style fast unification is unlikely to happen in Korea because DMZ is dense military region and North government keep monitoring North Korean residents' illegal migration toward the South. From another point of view, Korea have enough time to prepare for unification.

[10]Bank of Korea, Economic Statistics System, ECOS

[11]Kim and Roland(2012), Scenarios for Transition to a prosperous Market Economy in North Korea, p.517

3 Overview of North and South Korean Economy

Before we go on main model, we will take a look into an overview of North and South Korean economy. Having such a knowledge is not only intuitively helpful to image their economic status quo but also necessary to obtain proper parameters in mathematical approach we will use in the next chapter. Especially we will examine some important economy variables - Population, GNI per capita, Education, Infrastructures.

3.1 Population

Table 1: Population of North and South Korea (Unit : thousand)

	1990	1995	2000	2005	2010	2012
North(a)	20,221	21,715	22,702	23,561	24,187	24,427
South(b)	42,869	45,093	47,008	48,138	49,410	50,004
Ratio(a/b)	0.47	0.48	0.48	0.49	0.49	0.49

Source : Bank of Korea, Economic Statistics System, ECOS.

As you see in Table 1, North and South Korea population have increased consistently since 1990. South Korea population was about 43 millions in 1990 and it has increased slightly more than 50 millions in 2012. North Korea population was about 20 millions in 1990 which is half of its counterpart and more than 24 millions in 2012. Based on this quite a few stable ratio of South Korea population to North Korea population - approximately 0.49, we assume $\lambda \equiv \frac{L_n}{L_s} = 0.49$ where L_s = South Korea population and L_n = North Korea population.

3.2 GNI per capita

Table 2: GNI per capita of North and South Korea (Unit : dollars)

	1990	1995	2000	2005	2010	2012
North(a)	810	790	840	1,050	1,240	1,370
South(b)	4,460	9,050	12,770	17,960	23,780	25,590
Ratio(a/b)	0.18	0.08	0.07	0.06	0.05	0.05

Source : Bank of Korea, Economic Statistics System, ECOS.

In Table 2, we consider the most important economic variable in this thesis - GNI per capita of North and South Korea. The ratio of GNI per capita of North to the South, which I denote as $\theta \equiv \frac{y_n}{y_s}$, where y_s = GNI per capita of South Korea and y_n = GNI per capita of North Korea, has increased for more than 20 years in the table. Unlike λ in Table 1, θ varies over time so extra work is needed for setting value of θ. We will take more time in estimating θ in Chapter 5. In passing, fiscal burden of South Korean government can be measured by making θ to be 1 which means North Korean people have same amount of income compared to South Korean people after unification.

Table 3: Growth rate of North and South Korea

	1990	1995	2000	2005	2010	2012
North	−4.3%	−4.4%	0.4%	3.8%	−0.5%	1.3%
South	9.3%	8.9%	8.8%	4.0%	6.3%	2.0%

Source : Bank of Korea, Economic Statistics System, ECOS.

Table 3 shows growth rate of North and South Korea. As we already know, South Korea has developed fast, approximately 8 9% in 1990s and the growth rate landed down to 2% - 3% in the early 2010s. In contrast, it does not seem to increase over the same period in North Korea. In fact, it decreased in 1990s and just over 1% in 2012.

3.3 Education

Table 4: Number of students in North and South Korea (Unit : thousand)

	School	1997	2000	2005	2010	2012
North	Element	1,830	1,631	1,376	1,500	1,500
	Middle	2,824	2,278	2,397	2,200	2,200
	Univ	310	310	530	510	510
South	Element	3,784	4,020	4,023	3,299	2,952
	Middle	4,517	3,932	3,774	3,937	3,769
	Univ	2,267	2,829	3,020	3,134	3,222

Source : Korean Statistical Information Service, KOSIS, http://kosis.kr/

Table 4 represents number of students at each educational level in the two countries. It is quite surprising that education level of North Korea in terms of elementary and middle school enrollment is not low, rather slightly higher than that of South, given population ratio. In 2012, there exists 2.9 million elementary school students and 3.7 million middle school students in the South and 1.5 million elementary school students and 2.2 million middle school students in the North. Considering population ratio of 0.5, it seems that North Korea has more human capital than the South. However, when we consider University enrollment, South Korea posesses definetely higher level of human capital. Since North and South Korea share large portion of common denominator such as race, nationality, historical background and culture, I believe that South Korea's high University enrollment never come from other factor but income. Because North Korea people can not afford University tuition fee, they have low human capital. That is why I considered "Learing by Doing Model" in this analysis which means that level of human capital is a function of level of capital.

3.4 Infrastructure

Table 5: Infrastructure status in North and South Korea

		1990	1995	2000	2005	2010	2012
North	Car	1.3	0.9	0.7	0.5	0.4	0.4
	Cement	613	422	460	593	627	644
	Iron	336	153	108	116	127	122
	Ship	54	90	85	90	80	76
	Road	2,300	2,333	2,363	2,549	2,595	2,611
	Port	3,490	3,501	3,550	3,700	3,700	3,700
South	Car	132	252	311	369	427	456
	Cement	3,357	5,513	5,125	4,719	4,742	4,708
	Iron	2,312	3,677	4,310	4,782	5,891	6,907
	Ship	711	633	615	1,007	1,427	*N/A*
	Road	5,671	7,423	8,877	10,229	10,556	10,570
	Port	22,432	28,520	43,044	65,033	83,002	*N/A*

note : Car - 10 thousand, Cement - 10 thousand ton, Iron - 10 thuosand ton, Ship - 10 thousand ton, Road - km, Port - 10 thousand ton. *Source* : Bank of Korea, Economic Statistics System, ECOS.

Table 5 represents infrastructure status of the two countries. Many categories can be considered as one country's infrastructures however, in this Table 5, I consider only 6 important and accessible data categories - number of cars, cement production, number of ships, total length of road, port loading capacity. As you see, North and South Korea have big infrastructure gap. In the model used in this thesis, infrastructure is very important

factor as they determine labor productivity and finally determine GNI per capita. That is why South Korea spends large amount of government transfer into North Korean infrastructure. So far, we examined North and South Korea's economic overviews such as population, GNI per capita, education status and infrastructure. As a result, we obtained important parameters λ, θ and economic idea of 'Learning by doing' for human capital. We are now ready to jump into mathematical analysis.

4 The Model Setup

4.1 Firms

We first start with a large number of competitive firms and each firm's behavior follows general CRS Cobb-Douglas product function with human capital as in equation (1). Notation $i = S, N$ represents the South and the North respectively. Prices are normalized to one. Production function, law of motion of capital and human capital equations are as follows

$$Y_i = A_i {K_i}^{\alpha_1} {H_i}^{\alpha_2} {L_i}^{1-\alpha_1-\alpha_2} \quad (1)$$

$$\dot{K}_i = I_i - \delta K_i \quad (2)$$

$$H_i = Z_i {K_i}^{\Omega} \quad (3)$$

Physical capital evolves as equation (2) without losing generosity, and human capital is a function of physical capital K and human capital productivity Z as in Learning by doing model in Lucas(2004). we assume Ω to be 1 for simplicity. Plugging (3) into (1), we obtain following equation.

$$Y_i = A_i \Psi_i {K_i}^{\alpha} {L_i}^{1-\alpha} \quad (4)$$

Where use is made of the fact that ${Z_i}^{\alpha_1} = \Psi_i$ and ψ represents human capital productivity. Using F.O.C of $MPL = w$ and $MPK = r$, we obtain following equation (4) and (5).

$$(1-\alpha) A_i \Psi_i \left(\frac{K_i}{L_i}\right)^{\alpha} = w_i \quad (5)$$

$$\alpha A_i \Psi_i \left(\frac{K_i}{L_i}\right)^{\alpha-1} - \delta = r_i \quad (6)$$

4.2 Government

Government receives revenue as the amount of tax ratio τ times income Y. So government expenditure equations follows equation (7). Beware that in this paper notation G represents stock of infrastructure(physical and human capital both), not an ordinary donation for government spending as a flow. q denotes the share spent on infrastructure, δ denotes ratio of depreciation, Z denotes regional redistribution and x denotes the share of South Korean tax revenues transferred to the North.

$$\dot{G}_i = q_i \tau Y_i + h_i \tau Y_i - \delta G_i \tag{7}$$

$$Z_s = (1 - q_s - h_s)\tau Y_s - x\tau Y_s \tag{8}$$

$$Z_n = (1 - q_n - h_n)\tau Y_n + x\tau Y_s \tag{9}$$

Productivity A_i and human capital productivity ψ_i is function of A and ψ, which are constant intrinsic productivity respectively. $\frac{G_i}{L_i}$ represents infrastructure stock per capita. ν measures ratio of how much infrastructure stock per capita contribute to physical productivity and human productivity.

$$A_i = A\left(\frac{G_i}{L_i}\right)^{(1-\alpha)\nu} \tag{10}$$

$$\psi_i = \psi\left(\frac{G_i}{L_i}\right)^{(1-\alpha)(1-\nu)} \tag{11}$$

Equation (4) can be re-expressed using (10) and (11) then we obtain equation (12) which will be useful in equation calculation later.

$$Y_i = A\psi\left(\frac{G_i}{K_i}\right)^{(1-\alpha)} K_i \tag{12}$$

Insert (10) and (11) into (6) then use interest rate parity $r_s = r_n$ and then we obtain (13). But here we consider risk premium in North Korea.

κ measures risk premium which means that capital \$1 input ends up \$$k$ output because of risk premium in the North. This assumption is realistic because of inefficiency in North Korea economy due to legacy of communism regime even after unification.

$$\alpha A\psi \left(\frac{G_s}{L_s}\right)^{(1-\alpha)} - \delta \quad = \quad \kappa\alpha A\psi \left(\frac{G_n}{L_n}\right)^{(1-\alpha)} - \delta \tag{13}$$

Organizing both sides of (13), we get following equation (14).

$$\kappa^{1/(1-a)} \left(\frac{G_n}{G_s}\right) \quad = \quad \left(\frac{K_n}{K_s}\right) \tag{14}$$

4.3 Households

As in Firms, there exits many Households each maximizing utility function following equation (15). This is a general Constant Relative Risk Aversion - CRRA utility function. Each household supply one unit of labor for simplicity. ρ represents time preference rate and c denotes consumption per capita. $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution

$$U_i = \int_0^\infty \frac{{c_i}^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \tag{15}$$

The budget constraint is as follows

$$c_i + \dot{a_i} = (1-\tau)(r_i a_i + w_i) + z_i \tag{16}$$

The left side of (16) represents expenditure on consumption and savings since a denotes capital stock per capita. The right side represents after tax income plus redistribution from government. I used current value Hamilto-

nian[12] for this simple dynamic optimization.

$$H = \frac{{c_i}^{1-\sigma} - 1}{1-\sigma} + \epsilon((1-\tau)(r_i a_i + w_i) + z_i - c_i) \tag{17}$$

$$\frac{dH}{dc_i} = {c_i}^{-\sigma} = \epsilon \Leftrightarrow \frac{\dot{c_i}}{c_i} = -\frac{\dot{\epsilon}}{\sigma\epsilon} \tag{18}$$

$$\dot{\epsilon} = \rho\epsilon - (1-\tau)r_i\epsilon \tag{19}$$

Finally we obtain equation (20) which is an optimal path for consumption given τ, r_i, ρ and σ.

$$\frac{\dot{c_i}}{c_i} = \frac{(1-\tau)r_i - \rho}{\sigma} \tag{20}$$

4.4 Abroad

In scenario 3, we will consider capital inflow from abroad into North Korea and it is same as that Korean government borrow same amount of money in the process of unification. However, in scenario 1 and 2, we ignore borrowing money from abroad. Assume that North Korea can borrow $B_0 = b_0 Y_0(0)$ at interest rate of i. Equation (21) means γ_b - rate of change of debt - income ratio - is equal to rate of change of debt minus rate of change of income.

$$\gamma_b \equiv \frac{db}{b} = \frac{dB}{B} - \gamma y_s \tag{21}$$

Assuming a constant repayment rate s, rate of change of B in (21) is given by $\dfrac{\dot{B}}{B} = -s$. The debt actually paid out at time t is defined by $\hat{B}(t)$ such that

$$B_0 = \int_0^\infty \hat{B}(t)dt = \int_0^\infty \bar{B}(t)e^{-\nu t}dt \tag{22}$$

[12]Refer to Mathematical Economics 4th edition p.651 for theoretical explanation and Lucas(2004) for practical examples

We consider ν as a policy variable which the government control how the burden of foreign debt is allocated over time.

$$\hat{b} = \nu \frac{B_0}{Y_0} e^{-(\nu - i)t - g(t)} \tag{23}$$

Where d is the debt charges in units of South Korean GDP. d is determined as follows : Outstanding debt decreases at rate s, i,e, debt charges at time t are $D = (i + s)B_0 e^{-st}$ and $d = D/Y_s$ are calculated according to

$$d(t) = (i + s)e^{-(st + g(t))b_0} \tag{24}$$

4.5 Convergence

Now we can use all equations we derived above in this convergence section. If GNI per capita of North and South tends to converge, θ will be 1 in the end by definition. Equation of θ as follows

$$\theta \equiv \frac{y_n}{y_s} = \frac{1}{\kappa} \frac{K_n/L_n}{K_s/L_s} = \kappa^{\alpha/(1-\alpha)} \frac{G_n}{G_s} \frac{1}{\lambda} \tag{25}$$

where

$$\lambda \equiv \frac{L_n}{L_s} \tag{26}$$

insert (10),(11) and (25) into (7) then we obtain law of motion of θ as follows

$$\gamma_\theta \equiv \frac{\dot{\theta}}{\theta} = \frac{\alpha}{1 - \alpha} \frac{\dot{\kappa}}{\kappa} + \tau A \psi \left(\frac{G_s}{K_s} \right)^{-\alpha} [(q_n + h_n)\kappa^{\alpha/(1-\alpha)} - (q_s + h_s)] \tag{27}$$

In so far as the North lags behind the South because of relatively row physical and human productivity, North Korean government have to spend

more money into infrastructures than that of the South. The policy rule is executed by a set of monotonous functions f such as

$$q_n = f(\theta)q_s, (f\prime < 0, f(1) = 0) \tag{28}$$

where

$$f(\theta) = a\left(\frac{1-\theta}{\theta}\right) + 1 \tag{29}$$

Similar law of motion of human capital mechanism is in (30)

$$h_n = f(\theta)h_s \tag{30}$$

However, unlike physical capital, human capital have spill over effect[13], which means that relative high level of human capital accumulation in the South can have positive effect on the North. For example, tacit knowledge about business management embodied in South Korean CEOs are definitely beneficial to the North Korean novice entrepreneurs. Therefore, considering spillover effect, we can represent (30) as in

$$h_n = f(\theta)h_s[\theta] + \frac{1}{\theta}h_s[1-\theta] = g(\theta)h_s \tag{31}$$

which is a convex combination of $f(\theta)h_s$, the amount of direct investment in the North, and $1/\theta h_s$, the spillover effect from the South, weighting θ and $1-\theta$.

We now consider a dynamic equilibrium equations. K denotes national capital stock after unification and it can be expressed in terms of only the South capital stock such as

$$K = K_s + K_n = (1 + \theta\lambda\kappa)K_s \tag{32}$$

[13]Richard Blundell et al(1999), Human Capital Investment: The Returns from Education and Training to the Individual, the Firm and the Economy.

To solve law of motion of capital stock, we obtain

$$\dot{K} = (1 - \tau q_s - \tau h_s)Y_s + (1 - \tau q_n - \tau h_n)Y_n - C - \delta K \tag{33}$$

Insert (12) into (33) and after manipulation, we have

$$\dot{K} = (1 - \tau q_s - \tau h_s)A\psi \left(\frac{G_s}{K_s}\right)^{1-\alpha} K_s +$$
$$(1 - \tau q_n - \tau h_n)A\psi \left(\frac{G_n}{K_n}\right)^{1-\alpha} K_n - C - \delta K \tag{34}$$

Finally the growth rate of the nation-wide capital stock can be expressed as

$$\gamma_K = \frac{\dot{K}}{K} = A\psi g_s^{1-\alpha}(1 + \lambda\theta\kappa)^{-\alpha}[1 + \theta\lambda - \tau q_s\{1 + \lambda\theta f(\theta)\}$$
$$-\tau h_s\{1 + \lambda\theta g(\theta)\}] - \chi - \delta \tag{35}$$

where $\chi \equiv C/K$ which means the economy-wide consumption - capital ratio and $g_s \equiv G_s/K$ which menas intrastructure of the South per unit of nation - wide private capital. Insulting (6) into (20), the growth rate of the consumption capital ratio is obtained as

$$\gamma_\chi \equiv \frac{\dot{C}}{C} - \gamma_K = \frac{1}{\sigma}[(1 - \tau)\{\alpha A\psi g_s^{1-\alpha}(1 + \theta\lambda\kappa)^{1-\alpha} - \delta\} - \rho] - \gamma_K \tag{36}$$

Using new notations, Egs. (7) and (27) can be re-expressed as (37) and (38).

$$\gamma_{g_s} \equiv \frac{\dot{G}_s}{G_s} - \gamma_K = (q_s + h_s)\tau A\psi g_s^{-\alpha}(1 + \lambda\theta\kappa)^{-\alpha} - \delta - \gamma_K \tag{37}$$

and

$$\gamma_\theta = \frac{\alpha}{1 - \alpha}\frac{\dot{\kappa}}{\kappa} + \tau A\psi g_s^{-\alpha}(1 + \lambda\theta\kappa)^{-\alpha}[q_s\{\kappa^{\alpha/(1-\alpha)}f(\theta) - 1\}$$
$$+h_s\{\kappa^{\alpha/(1-\alpha)}g(\theta) - 1\}] \tag{38}$$

Dynamics after unification are therefore summarized by three differential equations for θ, g_s and χ. Now let's take a look at risk premium. κ is function of θ.

$$\kappa = \theta^{\Upsilon} (\Upsilon \geq 0) \tag{39}$$

An equilibrium of complete convergence uniquely determines $\theta^* = 1$ from (38). Insertion of (37) into (36) provides the implicit function

$$\begin{aligned} 0 = F(g_s^*) = \frac{1}{\sigma}[(1-\tau)\{\alpha A \psi g_s^{*1-\alpha}(1+\lambda)^{1-\alpha} - \delta\} - \rho] \\ -(q_s + h_s)\tau A \psi g_s^{*-\alpha}(1+\lambda)^{-\alpha} + \delta \end{aligned} \tag{40}$$

Solving for g_s^* is provided in Matlab Code in Appendix II and graphical results are provided in Additional Data in Appendix I. Finally, χ^* is obtained from (35) and (36) as

$$\begin{aligned} \chi^* = A\psi g_s^{*1-\alpha}(1+\lambda)^{-\alpha}\{(1+\lambda-\tau q_s(1+\lambda) - \tau h_s(1+\lambda)\} \\ -A\psi g_s^{*1-\alpha}(1+\lambda)^{-\alpha}\frac{\alpha(1-\tau)(1+\lambda)}{\sigma} - \delta + \frac{(1-\tau)\delta + \rho}{\sigma} \end{aligned} \tag{41}$$

Wage income net of taxes and transfer is $(1-\tau)(1-\alpha)y_i + z_i$. After substituting transfers from above and using the definition of θ one obtains the relative North-South income ratio

$$\phi = \frac{[1-\alpha(1-\tau)]\theta - (q_n + h_n)\tau\theta + x\tau(Ls/Ln)}{[1-\alpha(1-\tau)] - (q_s + h_s + x)\tau} \tag{42}$$

Inserting the policy rule and the definition of λ and solving for x yields

$$x = max\left[0, \frac{\{[1-\alpha(1-\tau)](\phi-\theta) + [f(\theta)g_s + g(\theta)h_s]\tau\theta - \phi\tau(g_s+h_s)\}\lambda}{\tau(1+\lambda\theta)}\right] \tag{43}$$

Equation (43) determines government transfer from the South to the North over time after unification. Quantitative results of x is used to calculate unification costs. So far, we did not consider Abroad section to obtain (43). Now let's consider it

$$\begin{aligned}\frac{Z_n}{L_n} &= (1-q_n-h_n)\tau y_n + x\tau\frac{Y_s}{L_s}\frac{L_s}{L_n} \\ &\quad +\hat{b}\frac{Y_s}{L_s}\frac{L_s}{L_n} = (1-q_n-h_n)\tau y_n + (x\tau+\hat{b})\frac{y_s}{\lambda} \qquad (44)\\ \frac{Z_s}{L_s} &= (1-q_s-h_s-x)\tau y_s - dy_s \qquad (45)\end{aligned}$$

Same mechanism as in (42) with money borrowing this time.

$$\begin{aligned}\phi &= \frac{(1-\alpha)(1-\tau)y_n + Z_n/L_n}{(1-\alpha)(1-\tau)y_n + Z_s/L_s} \qquad (46)\\ &= \frac{[1-\alpha(1-\tau)]\theta - (q_n+h_n)\tau\theta + x\tau(Ls/Ln) + \hat{b}/\lambda + d}{1-\alpha(1-\tau)] - (q_s+h_s+x)\tau} \qquad (47)\end{aligned}$$

$$x = max\left[0, \frac{\{\Omega(\phi-\theta) + \Sigma\tau\theta - \phi\tau(g_s+h_s) - d\phi\}\lambda - \hat{b}}{\tau(1+\lambda\theta)}\right] \qquad (48)$$

where $\Sigma = [f(\theta)g_s + g(\theta)h_s]$ which is a 50 x 1 matrix and $\Omega = [1-\alpha(1-\tau)]$ which is a constant. This equation finally represents dynamics of fiscal burden of South Korea after unification to make $\phi = 1$ and we are going to use this parameter x to calculate unification costs at each scenarios.

5 Calibrations

5.1 Scenario 1

With our limited information and the North Korea's unexpected actions so far, it is hard to predict the future of North Korean regime whether they will prosperous as in Chinese communist party or collapse as in USSR. In a broad sense, however, I draw 3 big pictures related to Korean unification in the middle of complex reality and theoretical simplicity. Three scenarios are as follows. Scenario 1 is German style sudden unification forced by many North Korean residents. Risky current regime collapse suddenly and a quite number of North Korean people move to the South Korea. another possibility is that sudden power vacuum in North Korea which might lead to abrupt unification of Korea. Scenario 2 is status quo unification. South Korean government cooperate with the North such as Mount Kumkang tour business, Kaesung industrial complex business, etc then unify with sufficient preparation. Scenario 3 is North Korea's Chinese style open and reform unification. More explanations will be later on. Main points of scenario 1 is that South Korea's insufficient preparation for unification and unexpected sudden unification. Therefore, I considered it will happen in 2014, 2030 and 2050[14].

[14]To compare unification cost, we have to consider real value of money, not nominal value of money. Therefore, I discounted total unification costs of 2030 and 2050, so that we can compare them directly

Scenario 1 parameters are in Table 6 as follows.

Table 6: Scenario 1 parameters

Parameters	Values	Parameters	Values
$\theta(2014)$	0.053011	σ	2.92
$\theta(2030)$	0.044836	α	0.81
$\theta(2050)$	0.036817	A	$\sqrt{0.5}$
q_s	0.065	ψ	$\sqrt{0.5}$
h_s	0.142	τ	0.25
Γ	0.1	λ	0.49
a	1/3	δ	0.05
ϕ	1	ρ	0.026

First, let's take a look at left column parameters. θs are calculated assuming 2% growth rate of South Korea and 1% growth rate of North Korea[15]. As you see, the later unification, the more gap between the North and South which is definite result by assumption. Productive government spending except for social security transfers is about 0.20[16] Based on 2014 Korean government budget[17], human capital investment h_s is 0.142 and physical capital investment q_s is 0.065. Total sum of h_s and q_s is about 0.2 which is consistent with Park and OECD's findings. Γ is 0.1 in scenario 1 which means worst risk premium[18]. a is 1/3 which is half of that in

[15]based on growth rate history using Bank of Korea date from 1990 to 2012 GNI per capita

[16]According to Park(1998, Table 10.5) and OCDE(2001) estimated 0.17.

[17]Ministry of Strategy and Finance, 2014년 예산안.

[18]Estimates of the risk premium are 1, 0.5 and 0 in Funke and Strulik(2005). I use same values in this thesis

the benchmark German case[19] since we consider human capital as well as physical capital. I set ϕ to be 1 to calculate unification cost to make the North Korean wage income up to the South Koreans' wage income.

Now, let's look at second column parameters. Theses parameters are likely to follow conventional values. σ is 2.92 and α is 0.81. A and ψ are $0.5^{1/2}$ respectively, so that product of them is 0.5^{20}. τ is the South Korean government expenditure share of GDP[21] which is about 0.25 based on Bank of Korea data and Ministry of Strategy and Finance date in 2013. λ is 0.49 as estimated in Chapter 2 population section. As in conventional economic value, discount rate δ and time preference ρ is 0.05 and 0.026 respectively.

[19] Funke and Strulik(2000)

[20] Funke and Strulik(2005)

[21] Bank of Korea for 2013 GDP and Ministry of Strategy and Finance for 2013 government expenditure

5.2 Scenario 2

Scenario 2 is an unification after steady economic cooperation. To be specific, steady economic cooperation contains economic programs such as Kaeseong Industrial Complex, Kumgang Mountain tourism and Rajin-Hassan logistics partnership, DMZ peace park[22]. Successful execution of such projects will attract foreign capital and promote opportunities to introduce capitalism. Above all, forming trust between the North and South will be the most precious outcome, then finally it reduces unification costs after all.

Scenario 2 parameters are in Table 7 as follows.

Table 7: Scenario 2 parameters

Parameters	Values	Parameters	Values
$\theta(2030)$	0.053536	σ	2.92
$\theta(2050)$	0.053536	α	0.81
q_s	0.065	A	$\sqrt{0.5}$
h_s	0.142	ψ	$\sqrt{0.5}$
Γ	0.05	τ	0.25
a	1/3	λ	0.49
ϕ	1	δ	0.05
ρ	0.026		

Let's take a look at the most important criterion θ. GNI per capita in the North will grow faster than that under scenario 1, but not much. I assumed GNI per capita in the North will grow 2% annually until 2030 and 2050 unification time respectively due to economic help from the South, but no significant reduction in the South because the amount of help is

[22]KDI Review of the North Korean Economy(KDI 북한경제리뷰), May, 2014.

trivial compared to GDP scale of the South. Therefore, θ is slightly bigger than scenario 1. Benefits of economic cooperation is not just a growth of GDP, but also opportunity to learning business management. Therefore, I set γ to be 0.05 which is bad case rather than 0.1 worst case in scenario 1. Other parameters are same in scenario 1.

5.3 Scenario 3

This time we consider more rapid changes in the North. Based on China's successful economic reform and opening up experience, it is highly possible scenario that North Korea will decide to open to the world and accept capitalism by keeping communist political system intact as in China. If this surprising change goes successfully, unification cost of course will significantly be reduced since fiscal burden of the South will be less. However, incentives of unification, from a North Korea point of view, will be reduced as well for they have more power and money ever before. For analytical simplicity, we assume that net-benefits of unification are positive to both government, then unification happens even after the North's economic reform and opening up.

Scenario 3 parameters are in Table 8 as follows.

Table 8: Scenario 3 parameters

Parameters	Values	Parameters	Values
$\theta(2030)$	0.208408	σ	2.92
$\theta(2050)$	0.542734	α	0.81
q_s	0.065	A	$\sqrt{0.5}$
h_s	0.142	ψ	$\sqrt{0.5}$
i	0.03	τ	0.25
γ_y	0.03	a	1/3
Γ	0.00	ϕ	1
s	0.1	λ	0.49
v	0.1	δ	0.05
b	1/5(1/3)	ρ	0.026

First, let's examine θ. Fortunately, we have some empirical examples how fast they grow after economic reformation such as China and Vietnam cases. Based on previous experiences of other countries, I assume 10% growth rate until 2030 and 7% until 2050. So $\theta(2030)$ is about 0.2 and $\theta(2050)$ is about 0.54. γ is 0 in this scenario which means no risk premium after unification due to economic reform and opening up[23]. This scenario is faced with fundamentally different situation compared to previous 2 scenarios - capital inflow into the North from abroad. s is 0.1 and v is 0.1, b is $\frac{1}{3}$ for more boworring case and $\frac{1}{5}$ for less borrowing case. i is 0.03 by conventional wisdom and γ_y is 0.03. Other parameters are same as before.

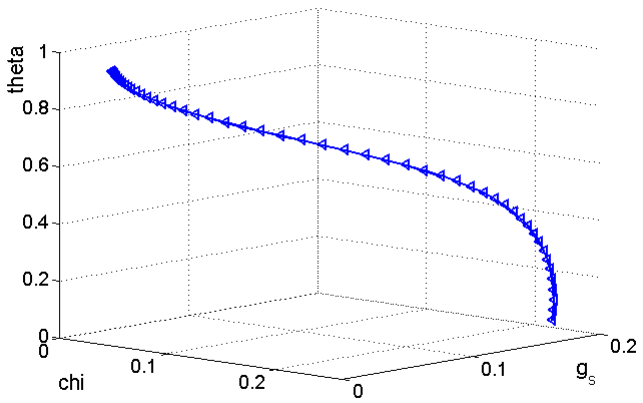
[23]Considering China's astonishing economic success, it is unrealistic assumption that risk premium exist in this scenario

6 Quantitative Results

6.1 Scenario 1

Using Matlab program[24], we finally arrived at graphical and numerical results at each scenario. These are as follows.

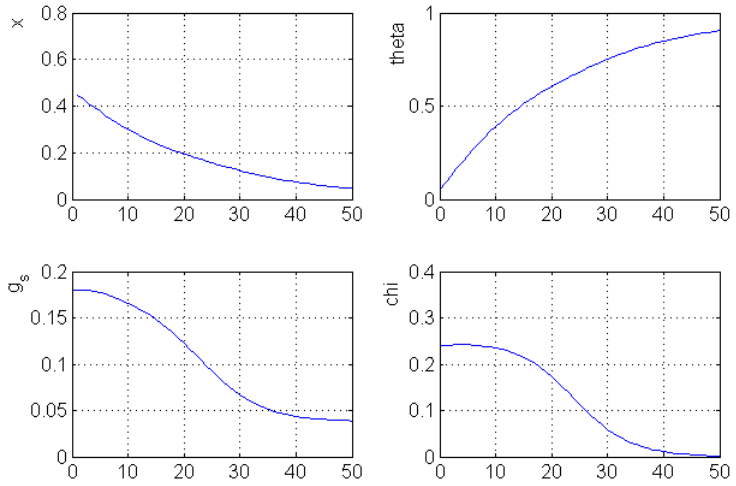
Figure 1: Optimal path after unification : Scenario 1, 2014



This 3D plot of arrowheaded line represents optimal path of θ, χ and g_s which are equation (36), (37) and (38) after unification at 2014 year under scenario 1. We only examine one representative case(2014yr case here) at each scenario. Time t is used as a parametric variable in this case. As t goes from 1 to 50 (50 years after unification) 3D coordinates of θ, χ and g_s are represented as a arrowheaded line in Figure 1. As you see, GNI per capita in the North catch up faster in the early period then slower in the late period as θ goes to 1.

[24]Partial Matlab code is provided in Appendix II

Figure 2: Optimal path of x, θ, g_s and χ : Scenario 1, 2014



Four 2D subplots in Figure 2 shows dynamics of x, θ, g_s and χ each after unification over 50 years. Of course these graphs are for 2014 year case only. graph of x represents at very early time, about 40% of South government tax revenue have to transfer into the North resident's to make wage income - ϕ equal to be 1. As I mentioned before, this will eliminate incentives to migrate to the South[25]. After 20 years later from unification date 2014, x reduces until 20% and it goes down under 10% after 50 years. This means government burden of the South will reduce over time as we already expected. θ shows that GNI per capita ratio converges to 1 fast over time. To achieve 50% of GNI per capita of the South, it only takes about 15 years

[25]Strictly speaking, terms of 'South' government and 'North' government do not make sense since there is only one unified Korea government, however, for analytic purpose we continue to divide into the South and the North.

after unification. 50 years later, it achieves about 90%.

Figure 3: Dynamics of unification costs : Scenario 1, 2014

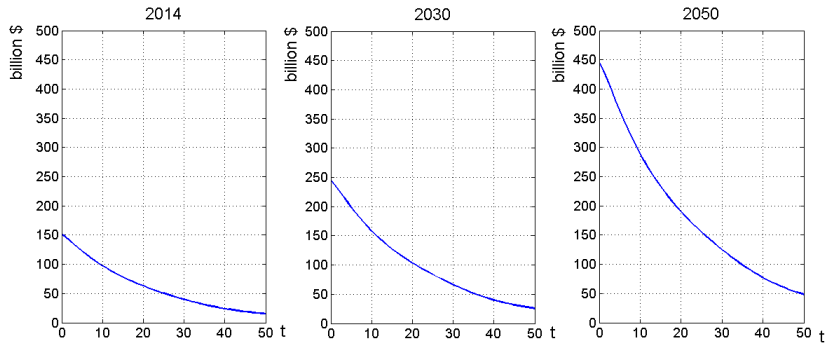
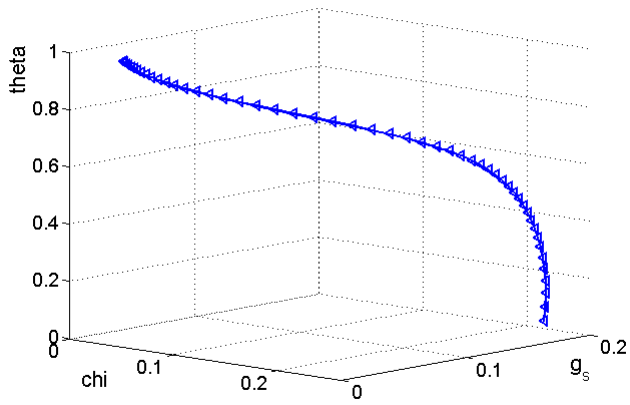


Figure 3 represents dynamics of unification costs at each unification date. We can obtain some important intuitions from this results. As a result, unfication costs are higher as we unify later. In 2014 it starts from slightly below \$150 billions and less later on, but it is more \$200 billions in 2030 case and about \$450 billions in 2050 case. This tells us that in scenario 1, best way to reduce unification costs is simply fast unification. This is simply because of θ. Bigger θ means huge fiscal burden of the South to make wage income equal to 1 after unification. Data generated from Matlab program are provided in Appendix I and total present value is provided in Table 9.

6.2 Scenario 2

Results under scenario 2 are quite similar to those in scenario 1 above.

Figure 4: Optimal path after unification : Scenario 2, 2030



In Figure 5, θ also converges fast over time and x decreases from 50% and goes down to about zero 50 years later. As we expect, it decreases faster than scenario 1. However, this results are not clearly distinguish by naked eyes. Other results are almost same as in scenario 1.

Figure 5: Optimal path of x, θ, g_s and χ : Scenario 2, 2030

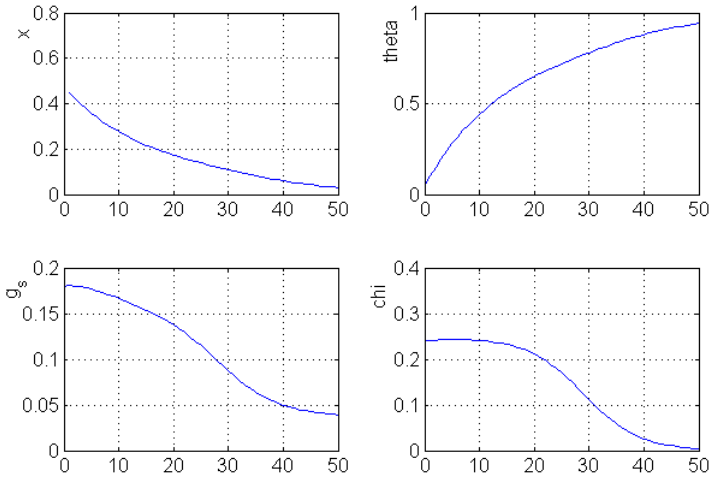
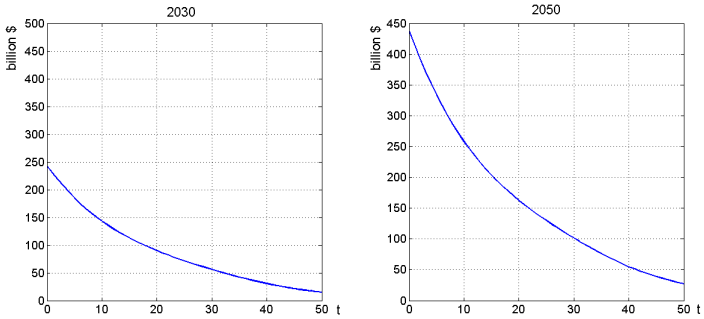


Figure 6 represents unification costs under scenario 2 in 2030 year case. It is about \$250 billions at the beginning in 2030 year and \$450 billions at the beginning in 2050 year case. Even though it is hard to notice with naked eyes, numerical results tell us that scenario 2 is cheaper than scenario 1. Table 9 show that under the same condition we can save about \$380 billions in scenario 2 in 2030 case in present value, \$500 billions in 2050 case in present value as well. Moreover, we can also find that 2030 year unification under scenario 2 is cheaper than 2014 year unification under scenario 1. Of course, this figures are dependent on parameters and assumptions, but can tell us economic intuitions how to reduce unification costs - time and economic cooperation between the North and the South are two key factors.

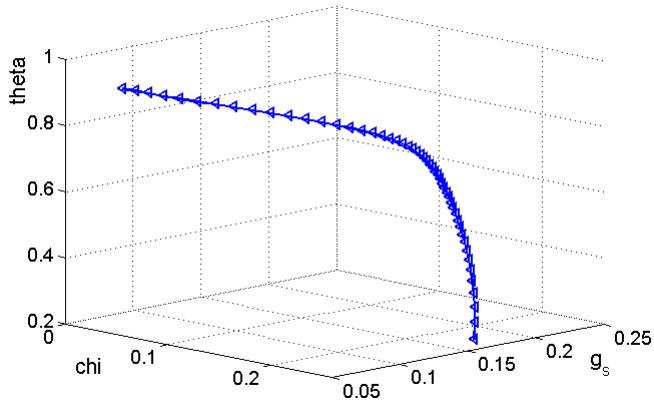
Figure 6: Dynamics of unification costs : Scenario 2, 2030



6.3 Scenario 3

From now on, we have broad distinctive results in scenario 3, As usual we start with 3D plot of optimal path after unification.

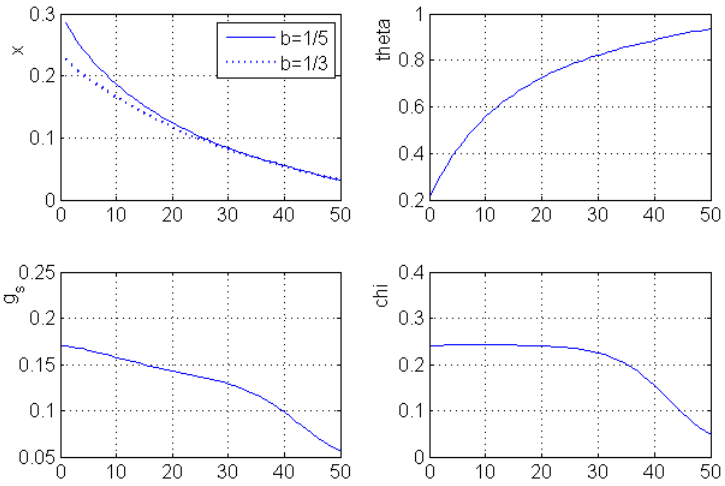
Figure 7: Optimal path after unification : Scenario 3, 2030



As before we start with this 3D plot of optimal path of θ, χ and g_s after unification in 2030 year under scenario 3. we only examine one representative case(2030 year case here). Time t is used as a parametric variable in this case. As t goes from 1 to 50 (50 years after unification) 3D coordinates of θ, χ and g_s are represented as a arrowheaded line in Figure 7. Judged by intervals of arrows along the opimal path line, we could say GNI per capita in the North catch up slowly in the early period then faster in the late period as θ goes to 1. However, this is not exactly right. θ goes to 1 faster than scenario 1 even in the early time and far faster in the late time due to no risk premium, high value of initial θ and capital inflow from abroad

in this optimistic scenario. We assumed 2 case 'more capital inflow case (b=1/3)' and less capital inflow case (b=1/5)'. Either way dose not affect optimal path of variables since equation (36), (37), (38) are all independent of capital inflow variable b. Instead, it affects government transfers, so we have 2 optimal paths of x in Figure 8 below.

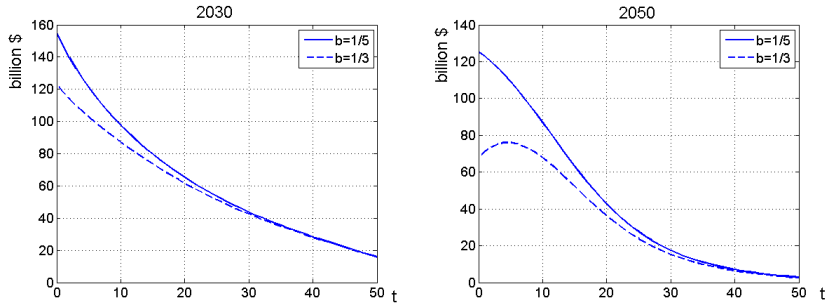
Figure 8: Optimal path of x, θ, g_s and χ : Scenario 3, 2030



In Figure 8, we can find rather distinct differences compared to it's counterparts Figure 2 and Figure 5 above. First, when it comes to x, it is much lower now. It starts from 30% or even slightly 20% if b=1/3 case. Then it decreases to 10% only 25 years both. The gap between dotted line (b=1/3) and solid line (b=1/5) can make significant unification costs. Calculated unification costs are provided in Appendix I. Second, let's take a look at θ. Based on scenario 3 assumptions, initial values of θs are bigger

than other scenarios, 0.21 and 0.5 for 2030 year case and 2050 year case respectively. More than that, it grows faster than other scenarios. As a result, slope of optimal path of θ is steeper in this scenario 3.

Figure 9: Dynamics of unification costs : Scenario 3, 2030



Considering previous results about unification costs, we can expect that unification costs will be reduced under unification scenario 3 and even more reduced with more capital inflow case (b=1/3). Time of unification is also key factor as well. Figure 6 proves all of our expectations are right. In 20130 year case, about $180 billions are needed then decreased monotonously until 2080 year (50 years later from 2030) with less capital inflow but it starts about $120 billions and same thereafter with more capital inflow case. Total unification costs are $3.23 trillions ($2.14 trillions in present discounted value based on 2014 price level) and $2.92 trillions ($1.94 trillions in present discounted value based on 2014 price level) respectively. Before we go on 2050 year case in scenario 3, let's remind that one simple rule - the sooner unification, the less costs. But, it is the other way around here!. The later unification, the less costs. It costs only $120 billions at the beginning and almost zero at 2100(50 years later from 2050 time case) with less capital

inflow and just $ 60 billions at the beginning then slightly increases and finally goes down to zero in 2100 with more capital inflow case. This is not difficult to understand this phenomenon. In this scenario, we assume the North's fundamental changes - reform and opening up and capital inflow and no risk premium thereby. These can not be overnight changes. As they take more time, then can reform and open more and more capital inflow and less risk premium after all. This is why we have less unification costs in the late time unification in scenario 3. Therefore, we can conclude very important rule about unification costs. Unification time is crucial but its effect is dependent on the North and South's situation. If only the South tries to help the North and the North does not change(scenario 1 and 2), early unification will reduce unification costs. If the North changes from inside(scenario 3) the late unification is less costly.

Total unification costs on each scenario are summarized in Table 9. Numbers in parenthesis means present discount values based on 2014 year. so we can directly compare unification costs each other.

Table 9: Estimated Korean Unification Cost(Present Value)

	Scenario 1	Scenario 2	Scenario 3
2014	$3.12(3.12)	-	-
2030	$5.09(3.37)	$4.51(2.99)	$3.23(2.14)/$2.92(1.94)
2050	$9.41(3.73)	$8.15(3.23)	$2.22(0.88)/$1.67(0.66)

Total unification costs on each scenario are summarized in Table 9. Numbers in parenthesis means present discount values based on 2014 year. so we can directly compare unification costs each other. Costs in each year are provided in Table 10 in Appendix I.

7 Conclusions

Now we can summarize our findings. In scenario 1, if the North and the South are unified, total $3.12 trillions are needed in 2014 unification time, $3.37 trillions and $3.73 trillions in present value, in 2030 and 2050 respectively. This tells us one simple rule - the sooner unification time the less unification costs. However, Scenario 1 is rather unrealistic considering current political and economic relationship between the North and the South. In scenario 2, which is more realistic scenario, it costs $2.99 trillions in 2030 unification time and $3.23 trillions in 2050 unification time. Compared to scenario 1, South Korean government can reduce substantial costs at same unification time. In Scenario 3, it costs from $2.14 trillions to $1.94 trillions in 2030 unification time and from $0.88 trillions to $0.66 trillions in 2050 unification time. From this results, we can conclude that late unification would reduce costs if the North is willing to change from inside. Surprisingly, $0.66 trillions are far lower than $3.12 trillions (2014 unification time in scenario 1!). This is a key point of this paper.

To reduce unification costs, the South try to induce the North to move forward by herself and be an independent agent of economic activity, rather then helplessly asking economic aid from the South. This is the best option the South has. Reducing costs by early unification is the second best option. Since not much empirical data are available when it comes to unification, South Korea have to keep in mind German's precious lessons. Politics without principle and insufficient preparation about unification could charge more burden on South Korean people's shoulders. Considering current economic situations it will be curse more than just burden.

As I mentioned earlier, previous studies about unification costs have serious drawback, from Economics point of view at least, which is lack

of strong economic theoretical background. In this paper, however, costs analysis is based on strict mathematics and Economic theories, even though there is no guarantee of better estimation in reality. Having theoretical strictness could be toxic at the same time. In real world, decision makings regarding unification procedures are rely on politics rather than economics. What if North Korean government change their attitude after they become rich in scenario 3? What if majority of South Korean people do not want to help the poor in the North? What if China and Russia involves in the middle of Korea unification process? These realistic situations are beyond Economics. Are then our efforts to reach findings in this paper useless? Not at all. It can shed some light on unknown area and finally give us right directions. Thanks to analytical approach, now we all know how to reduce unification. Knowing what to do is the purpose of this paper and how to do is another problem.

Estimated unification costs in terms of present values in this paper are quite similar to those in other papers despite each author's different methodology. Based on present value, variation of unification costs are from \$3.12 to \$0.66 trillions in this paper. Compared to estimated costs in Table 11, these results are within confidence interval. This enhance reliability of measurement of unification costs in this paper.

Considering various economic factors such as human capital, learning by doing, spill-over effect, unification time and unification scenarios, I believe this paper is distinct from previous two-region endogenous growth model. Leaving other researcher's work to improve and fix my rough ideas for better estimation of unification costs, I hope this paper somewhat contribute to extending our limited knowledge about unification costs.

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Appendix I : Additional Data

Figure 10: 3D plot for F(g_s*)

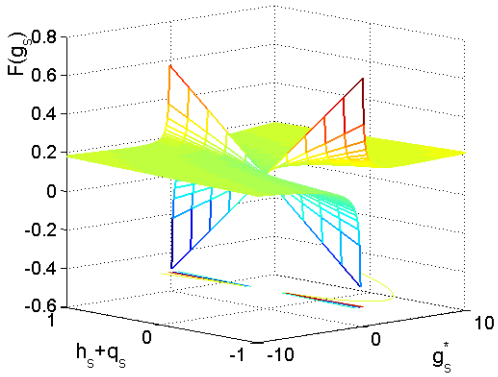

Figure 11: 2D plot for F(g_s*)

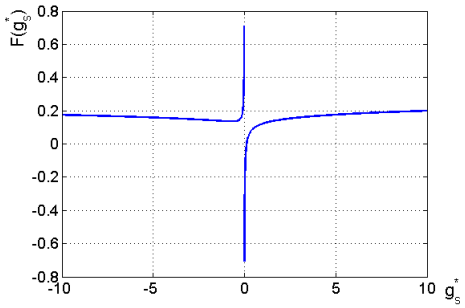

Table 10: Estimated annual government transfer (Unit : Billion dollars)

	Scenario 1			Scenario 2		Scenario 3	
Time	2014	2030	2050	2030	2050	2030	2050
0	151.34	244.96	446.14	242.72	438.39	155.14	125.35
1	145.85	236.41	431.29	230.20	415.77	146.87	122.74
2	139.99	227.12	414.78	217.88	393.52	139.45	119.78
3	134.02	217.56	397.60	206.16	372.35	132.78	116.48
4	128.10	208.05	380.38	195.15	352.47	126.65	112.90
5	122.36	198.75	363.50	184.86	333.88	120.99	109.04
6	116.83	189.80	347.19	175.28	316.57	115.76	104.95
7	111.55	181.24	331.57	166.34	300.43	110.80	100.65
8	106.53	173.11	316.72	158.00	285.37	106.17	96.18
9	101.78	165.39	302.62	150.23	271.34	101.81	91.56
10	97.28	158.09	289.29	142.94	258.18	97.63	86.85
11	93.02	151.19	276.71	136.14	245.89	93.66	82.06
12	88.99	144.65	264.80	129.76	234.36	89.93	77.26
13	85.17	138.49	253.55	123.73	223.48	86.35	72.48
14	81.55	132.64	242.95	118.07	213.25	82.90	67.76
15	78.12	127.09	232.89	112.75	203.64	79.60	63.14
16	74.84	121.83	223.35	107.70	194.52	76.46	58.65
17	71.72	116.82	214.33	102.91	185.87	73.48	54.34
18	68.74	112.04	205.73	98.39	177.70	70.60	50.21
19	65.87	107.47	197.53	94.07	169.90	67.82	46.30
20	63.12	103.08	189.71	89.95	162.46	65.16	42.61
21	60.46	98.86	182.21	86.02	155.36	62.62	39.15
22	57.89	94.79	175.02	82.24	148.54	60.19	35.92

Continued on next page...

	Scenario 1			Scenario 2		Scenario 3	
Time	2014	2030	2050	2030	2050	2030	2050
23	55.40	90.86	168.07	78.62	141.99	57.84	32.92
24	52.98	87.04	161.37	75.12	135.67	55.59	30.14
25	50.63	83.33	154.88	71.73	129.56	53.43	27.57
26	48.35	79.73	148.57	68.45	123.63	51.35	25.21
27	46.12	76.21	142.42	65.25	117.86	49.35	23.04
28	43.96	72.79	136.44	62.14	112.24	47.42	21.05
29	41.86	69.46	130.60	59.10	106.75	45.56	19.23
30	39.84	66.23	124.90	56.13	101.38	43.77	17.56
31	37.88	63.09	119.34	53.23	96.13	42.04	16.04
32	36.00	60.06	113.93	50.39	91.00	40.36	14.64
33	34.20	57.14	108.68	47.61	86.00	38.74	13.37
34	32.49	54.33	103.59	44.91	81.12	37.17	12.21
35	30.85	51.65	98.68	42.29	76.39	35.64	11.15
36	29.31	49.09	93.96	39.75	71.80	34.15	10.18
37	27.84	46.66	89.43	37.31	67.38	32.70	9.30
38	26.46	44.36	85.11	34.96	63.14	31.27	8.49
39	25.16	42.19	80.99	32.72	59.09	29.88	7.75
40	23.94	40.14	77.08	30.58	55.23	28.51	7.08
41	22.80	38.21	73.38	28.55	51.57	27.17	6.47
42	21.72	36.39	69.89	26.64	48.11	25.84	5.91
43	20.71	34.69	66.59	24.83	44.85	24.53	5.40
44	19.77	33.08	63.49	23.14	41.79	23.24	4.93
45	18.88	31.58	60.57	21.55	38.93	21.97	4.51
46	18.05	30.17	57.82	20.07	36.25	20.71	4.12
47	17.27	28.85	55.24	18.69	33.75	19.48	3.76

Continued on next page...

	Scenario 1			Scenario 2		Scenario 3	
Time	2014	2030	2050	2030	2050	2030	2050
48	16.53	27.60	52.82	17.40	31.43	18.28	3.44
49	15.84	26.43	50.54	16.20	29.26	17.10	3.14
50	15.19	25.33	48.39	15.08	27.24	15.96	2.87

Source : Author's own calculation.

Table 11: Estimated cost of Korean unification

Source	Methodology	Unification Date	Results
KDI(1991)	-	2000	$210 billions - $312 billions
KDI(1994)	Income Target	2000	$980 billions - $1 trillion
Noland et al(1996)	Income Target	2000	$415 billions - $983 billions
Noland et al(1996)	CGE Model	2000	$2.24 trillions(60%)
KDB(1994)	Income Target	1994-2000	$1.54 trillions(100%), $805 billions
Goldman Sachs(2000)	Income Target	2000-2010	$830 billions - $2.5 trillions
Goldman Sachs(2000)	Income Target	2005-2015	$1.7 trillions - $3.5 trillions
SERI(2005)	Itemized Budget	2015	$546 billions
신창민(2007)	Income Target	2015-2030	$860 billions - $1.3 trillions
Bank of Korea(2007)	Income Target	-	$500 billions - $900 billions
Beck(2010)	-	-	$2 trillions - $5 trillions
RAND(2005)	Income Target	-	$50 billions - $667 billions
FKI(2010)	Expert Survey	-	$350 trillions
김유찬(2010)	Itemized Budget	2010	$ 1.5 trillions - $ 2.2 trillions
HRI(2010)	Income Target	2010-2018	$157 billions -$706 billions
통일부(2011)	Itemized Budget	2020	$379.2 billions - $1.2 trillions
통일부(2011)	Income Target	2030	$813 billions - $2.8 trillions
통일부(2011)	Income Target	2040	$1 trillion - $3.2 trillions

Source : Song,Joonhyuk(2014) and 김구륜(2011)

Appendix II: Matlab Code

Only some part of Matlab code is provided. these are not the whole Matlab code.

```
% S3, 2050yr
clear all
% Setting Parameters

% S1 2014  = 0.05301167 , S1 2030 = 0.044836522 S1 2050 = 0...
    .036817641, S2 2030=2050=0.053536538, S3, 2030=0...
    .208408545, 2050=0.542734764
theta =0.542734764;
% Worst case 0.1 or Bad case 0.05 or Good case0.00
gamma = 0.00;

alpha = 0.81;
ramda = 0.49;
tau = 0.25;
Δ = 0.05;
rho = 0.026;
sigma = 2.92;
A = 0.5^(1/2);
psi = 0.5^(1/2);
phi = 1;
a = 1/3;
kappa = theta^gamma;
q_s = 0.065;
h_s = 0.142;

% Borrowing parameters
g = 0.03;
s=0.1;
```

```
v=0.1;
% More capital inflow case b=1/3, less capital inflow case ...
    b=1/5
b=1/3;
i=0.03;
```

```
% Solving for 'g_sstar'
syms x
solve((1/sigma)*((1-tau)*alpha*A*psi*x^(1-alpha)*(1+ramda)...
    ^(1-alpha)-(Δ+rho))-(q_s+h_s)*tau*A*psi*x^(-alpha)*(1+...
    ramda)^(-alpha)+Δ)
% Check a graph and conclude that g_sstar is ans(2)
g_sstar = ans(2)


% Graphing for 'F(g_sstar)'
x = [-10:0.1:10]; %g_s
y = [-1:0.2:1]; % h_s+g_s
% h_s+g_s can not be negative by defination. however, we ...
    consider sysmetric interval here.

% Calculates the necessary grid
[X,Y]=meshgrid(x,y);
% Calculates z and avoids a null denominator adding 'eps'
% (eps is the least possible number in Matlab)
Z =(1/sigma)*((1-tau)*(alpha*A*psi*X.^(1-alpha)*(1+ramda)...
    ^(1-alpha)-Δ)-rho)-(Y.*tau*A*psi).*(X.^(-alpha))*(1+...
    ramda)^(-alpha)+Δ+eps;
% Generates the second figure using 'meshc' to include the ...
    contour in the figure, and rotates the figure with 'view...
    '

% Graphing for g_sstar
```

```
syms q
q= -10:0.01:10;
r=(1/sigma)*((1-tau)*(alpha*A*psi*q.^(1-alpha)*(1+ramda)...
    ^(1-alpha)-Δ)-rho)-(q_s+h_s)*tau*A*psi*(q.^-alpha)*(1+...
    ramda)^(-alpha)+Δ+eps;
plot(q,r);

% Creating 3D graph for F(g_s^*)
subplot(1,2,1)
meshc(X,Y,real(Z))
xlabel('g_s^*')
ylabel('h_s+q_s')
zlabel('F(g_s^*)')
view([-42.5 10])

% Creating 2D graph for F(g_s^*)
subplot(1,2,2)
plot(q,r)
xlabel('g_s^*')
ylabel('F(g_s^*)')
grid on
```

```
% solve for g_s0
g_s0 = g_sstar*(1+ramda)/(1+theta*ramda*kappa)


% solve for chi0
chi0 = A*psi*(1+ramda)^(-alpha)*g_sstar^(1-alpha)*((1+ramda...
    -tau*q_s*(1+ramda)-(1+ramda)*tau*h_s)-(alpha*(1-tau)*(1+...
    ramda))/sigma)-(Δ*(1-tau)+rho)/sigma
```

```
% Create tspan. This means 50 years from unification.
```

```
tspan = 0:50;
% Inits [chi0 g_s0 theta]
% Setting tolerence
options=odeset('RelTol','1e-6','stats','on');
y0=[chi0, g_s0,theta];
% Use ODE23s function since these equations are stiff ...
    differential equations
[t, y] = ode23s(@finally3mfile, tspan, y0);

% Generating 3 column vectors from ODE solutions.
yy1=y(:,1);
yy2=y(:,2);
yy3=y(:,3);

% Creating 3D optimal path of 3 parameters after ...
    unification.
subplot(2, 2, 1);
plot3(yy1, yy2, yy3);
grid on;
xlabel('chi');
ylabel('g_s');
zlabel('theta');
view([42.5 12]);

% Creating 2D optimal path of theta after unification.
subplot(2, 2, 2);
plot(t, yy3);
xlabel('t');
ylabel('theta');
grid on;

% Creating 2D optimal path of g_s after unification.
subplot(2,2,3);
plot(t,yy2);
xlabel('t');
```

```
35 ylabel('g_s');
36 grid on;
37
38 % Creating 2D optimal path of chi after unification.
39 subplot(2,2,4);
40 plot(t,yy1);
41 xlabel('t');
42 ylabel('chi');
43 grid on;
```

```
1  function dy = finally3mfile(t,y)
2
3  %S1 2014  = 0.05301167 , S1 2030 = 0.044836522 S1 2050 = 0...
       .036817641, S2 2030=2050=0.053536538, S3, 2030=0...
       .208408545, 2050=0.542734764
4  theta = 0.542734764;
5
6  %worst case 0.1 or bad case 0.05 or good case0.00
7  gamma = 0.00;
8
9  alpha = 0.81;
10 ramda = 0.49;
11 tau = 0.25;
12 Δ = 0.05;
13 rho = 0.026;
14 sigma = 2.92;
15 A = 0.5^(1/2);
16 psi = 0.5^(1/2);
17 phi = 1;
18 a = 1/3;
19 kappa = theta^gamma;
20 q_s = 0.065;
21 h_s = 0.142;
```

```
22
23
24 %y(1)=chi, y(2)=g_s y(3)= theta
25 dy = zeros(3,1);
26 % Differential eqations for chi
27 dy(1) = (y(1)/sigma)*((1-tau)*(alpha*A*psi*y(2)^(1-alpha)...
       *(1+ramda*y(3)*y(3)^gamma)^(1-alpha)-Δ)-rho)-(1+ramda*y...
       (3)-tau*q_s*(1+ramda*y(3)*(a*(1-y(3))/y(3)+1))-tau*h_s...
       *(1+ramda*y(3)*(a*(1-y(3))+y(3)+(1-y(3))/y(3))))*A*psi*y...
       (2)^(1-alpha)*(1+ramda*y(3)*y(3)^gamma)^(-alpha)*y(1)+y...
       (1)*y(1)+Δ*y(1);
28 % Differential eqations for g_s
29 dy(2) = (q_s+h_s)*tau*A*psi*y(2)^(1-alpha)*(1+ramda*y(3)*y...
       (3)^gamma)^(-alpha)-y(2)*Δ-(1+ramda*y(3)-tau*q_s*(1+...
       ramda*y(3)*(a*(1-y(3))/y(3)+1))-tau*h_s*(1+ramda*y(3)*(a...
       *(1-y(3))+y(3)+(1-y(3))/y(3))))*A*psi*y(2)^(1-alpha)*(1+...
       ramda*y(3)*y(3)^gamma)^(-alpha)*y(2)+y(1)*y(2)+Δ*y(2);
30 % Differential eqations for theta
31 dy(3) = y(3)*(1-alpha)/(1-alpha-alpha*gamma)*(y(3)^(gamma*...
       alpha/(1-alpha))*((1+a*(1-y(3))/y(3))-1)*q_s+h_s*((a*(1-...
       y(3))+y(3)+(1-y(3))/y(3))*y(3)^(gamma*alpha/(1-alpha))...
       -1))*tau*A*psi*y(2)^(-alpha)*(1+ramda*y(3)*y(3)^gamma)...
       ^(-alpha);
```

```
1 % Solving for bhet
2 bhet = v*b*exp(-(v-i)*t-g*t)
3
4 % Solving for d
5 d=b*(i+s)*exp(-(s*t+g*t))
6
7 % Solving for x
8 % x is a 50 x 1 vector
9 thet = y(:,3);
```

```
fthet = (a*(1-thet)./thet)+1;
gthet = a*(1-thet)+thet+(1-thet)./thet;
x=max(0,((((1-alpha*(1-tau))*(phi-thet)+(fthet*(q_s+h_s))*...
    tau.*thet-phi*tau*(q_s+h_s)-d*phi)*ramda)-bhet)/(tau*(1+...
    ramda*phi)));

% Creating 2D time path for x after unification
plot(x);
grid on;
xlabel('t');
ylabel('x');
```

```
format long g
% Calculating fiscal burden based on government ...
    transferring
S5 = x.*3.9343*tau

% Summing total burden
total5  = sum(S5)
```

```
% Calulating present discounted value for b=1/3 case.
pv3 = total3*(1/(1+0.026))^16
pv5 = total5*(1/(1+0.026))^36

% Calulating present discounted value for b=1/5 case.
pv32 = total32*(1/(1+0.026))^16
pv52 = total52*(1/(1+0.026))^36

% Adjusting dollar units from trillion to billion for b=1/3...
     case.
graph3 = S3*1000
graph5 = S5*1000
```

```
12
13
14 % Adjusting dollar units from trillion to billion for b=1/3...
       case.
15 graph32 = S32*1000
16 graph52 = S52*1000
17
18 % Graphing unification cost for 2030yr case.
19 subplot(1,2,1)
20 plot(t,graph3)
21 ylabel('billion $')
22 xlabel('t')
23 title('2030')
24 grid on
25 hold on
26 plot(t,graph32,'--')
27 legend('b=1/5', 'b=1/3')
28
29
30 % Graphing unification cost for 2050yr case.
31 subplot(1,2,2)
32 plot(t,graph5)
33 ylabel('billion $')
34 xlabel('t')
35 title('2050')
36 grid on
37 hold on
38 plot(t,graph52,'--')
39
40 legend('b=1/5', 'b=1/3')
```

국문 초록

남북한 통일비용을 줄이는 방안 : 인적자본 고려한 두 지역간 내생적 성장모형을 중심으로

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본 논문의 목적은 남북한 통일비용을 추정하고, 이를 줄이는 방안에 대한 경제학적 함의를 얻고자 함이다. 통일비용은 인적자본을 고려한 두 지역간 내생적 성장모형을 이용하여 남한에서 북한으로의 정부 이전지출의 50년간 동학을 현재가치로 계산하였다. 독일 통일의 경험을 고려하고 남북한의 현재 상황을 감안하여 3개의 통일시나리오와 통일시점을 설정하였고, 각각의 통일비용을 추정 후 비교하였다. 시나리오 1은 독일식 흡수통일이며 시나리오 2는 현 상황에서 남북한 경제협력 후 통일이고 시나리오 3은 북한의 개혁개방 후 자본유입을 고려한 통일이다. 시점은 갑작스런 통일시점 2014년과 비교적 빠른 통일 2030년, 비교적 늦은 통일 2050년을 설정하였다. 결과적으로 남한의 북한에 대한 협조적인 자세와 북한의 개혁개방을 통한 내적변화 하에서 비교적 늦은 통일인 2050년 시나리오 3이 통일비용을 가장 크게 감소시키는 결과를 가져왔다.

주요어 : 남북한 통일비용, 내생적 성장모형, 인적자본, 북한, 통일비용.

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